

Synergistic combination of multiple nonlinear energy sinks in a large-scale building structure subjected to seismic ground motion

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Abstract. Traditional passive attachments, such as tuned mass dampers (TMDs), have been shown to be effective at reducing the response of structures subjected to stationary loadings, such as wind; however, these devices have failed to gain acceptance for attenuating the response of structures subjected to seismic ground motion. This paper presents the results of experimental shake table testing, using seismic ground motion, of a large nine-story structure equipped with two different types of innovative new mass dampers, known as nonlinear energy sinks (NESs). These devices use intentional and repeatable essential nonlinearities, impact in one case and smooth in the other, to couple the modes of an underlying nine-story structure and facilitate the targeted transfer of energy from lower modes of the structure to the higher modes, where it is dissipated rapidly. By measuring the system response with different ground-motion scaling factors when these devices are engaged both together and individually, the input amplitude dependence of the effectiveness of these devices is explored along with the synergistic effects realized by the combination of these two NES types. The results of this study show that, for the ground motion examined, the NESs studied can significantly attenuate the response of the base structure and reduce the peak demand on the structure. Furthermore, the synergistic effects realized by the simultaneous use of the different types of NES allows for consistent performance to be maintained across a broad range of ground motion amplitudes.

Keywords: nonlinear energy sink; nonlinear dynamics; passive control; mass damper

1 INTRODUCTION

In large parts of the world earthquakes pose a major threat to building structures and those who occupy them. To help mitigate this threat, numerous structure control devices have been proposed. One of the most popular of structural control devices currently in use for mitigating stationary loads is the tuned mass damper (TMD). Despite this popularity for stationary loading, TMDs have failed to gain widespread acceptance for use in mitigating seismic loading. Part of the reason for this is the inability of the TMD to reduce the peak demand on the system when subjected to a ground motions containing significant impulsive content (Chen and Wu 2003; Sladek and Klingner 1980). Additionally, the use of TMDs for mitigating the seismic response of structures has been questioned due to the substantial reduction in performance that can be observed when changes occur in the natural frequency of the structure the TMD is attached to (Lukkunaprasit and Wanitkorkul 2001; Pinkaew et al. 2003).

In recent years the ability of nonlinear energy sinks (NESs) to passively control the response of structures subjected to impulsive loads has been studied (Sapsis et al. 2012; Schmidt and Lamarque 2010; Al-Shudeifat et al. 2013; Vaurigaud et al. 2011; Wierschem et al. 2012). NESs are a type of passive mass damper; however, unlike traditional linear mass dampers, NESs utilize elements that produce an essentially nonlinear (non-linearizable) restoring force. With this essentially nonlinear restoring force, the response of an NES is not dominated by a single natural frequency and has the

ability engage in resonant vibration with the primary structure across a broad frequency range. Thus, the NES can vibrate with any mode of an underlying structure and participate in Targeted Energy Transfer (TET) (Kerschen et al. 2008), the transfer of energy from lower modes of vibration to higher ones where it can be dissipated more quickly. While extensive work has been done on assessing the ability of these devices to attenuate generalized impulsive loads, little research has been performed on investigating their effectiveness at attenuating seismic loading (Gourdon et al. 2007; Nucera et al. 2007)

In this paper, the effectiveness of a system of NESs at controlling the response of a large-scale base structure subjected to seismic ground motion is experimentally evaluated. The system of NESs examined in this work consists of a combination of multiple types of NESs which utilize smooth and non-smooth essential nonlinearities. In examining this system's effectiveness, particular attention is paid to the synergistic effects of utilizing multiple types of NESs.

2 EXPERIMENTAL SYSTEM

2.1 Base structure

The large-scale nine story base structure used in this experimental work to evaluate the effectiveness of a system of NESs at attenuating seismic loading is shown Figure 1. This structure is 5.13 m tall and has a mass of approximately 11,000 kg. The structure was constructed out of steel columns and steel plates. The dimensions of the base plate are 2.90 m by 1.22 m, while the floor plates are 2.74 m by 1.22 m. The base plate and the lowest seven floor plates are 3.81 cm thick and the top two floors plates are thick 4.44 cm. The top two floor plates have cut-outs in them to accommodate the NESs, which are discussed in detail in Section 2.2.

As shown in Figure 1, at each level, eight identical columns vertically connect the adjacent floor plates of the base structure. The cross-sectional dimensions of these members are 19.05 cm by 1.43 cm for the first floor columns and 13.97 cm by 1.43 cm for all other columns. At each level the columns are arranged in a 4 by 2 grid to form one bay in the short direction of the floor plates and three bays in long direction. These columns are oriented such that their stiff direction is in the long direction of the floor plates. This orientation, along with the high aspect ratio of the columns' cross section, results in the base structure being much softer in the short direction of the floor plates compared to the long direction. This asymmetry in the base structure was intentional as the structure was designed to be uniaxially loaded in the short direction of the floor plates. Additionally, as relatively large elastic displacements were desired, high strength steel was used to manufacture the columns.

As shown in Figure 1, cladding is attached to one side of the structure in the long direction of the floor plates. This cladding is composed of multiple 0.13 cm thick steel sheets that are attached with bolts to the sides of the floor plates. This cladding is incorporated into the base structure to accumulate a pressure loading during a separate set of tests which was designed to evaluate the effectiveness of the system of NESs at attenuating the global response of the structure resulting from to blast loading (Wierschem et al. 2013).

The modal parameters of the base structure were determined using data from instrumented hammer tests. As these tests were intended to determine the properties of the underlying linear structure, the NESs were locked. The results of this testing identified the following natural frequencies of the structure in its soft direction: 1.74, 5.37, 9.10, 12.72, 15.96, 18.95, 21.63, 23.92, and 25.48 Hz. These natural frequencies are similar to those of a typical midrise steel structure (American Society of Civil Engineers. 2010). Additionally, the torsional natural frequencies of the structure less than 30 Hz were determined to be 5.81, 18.16, and 29.36 Hz.

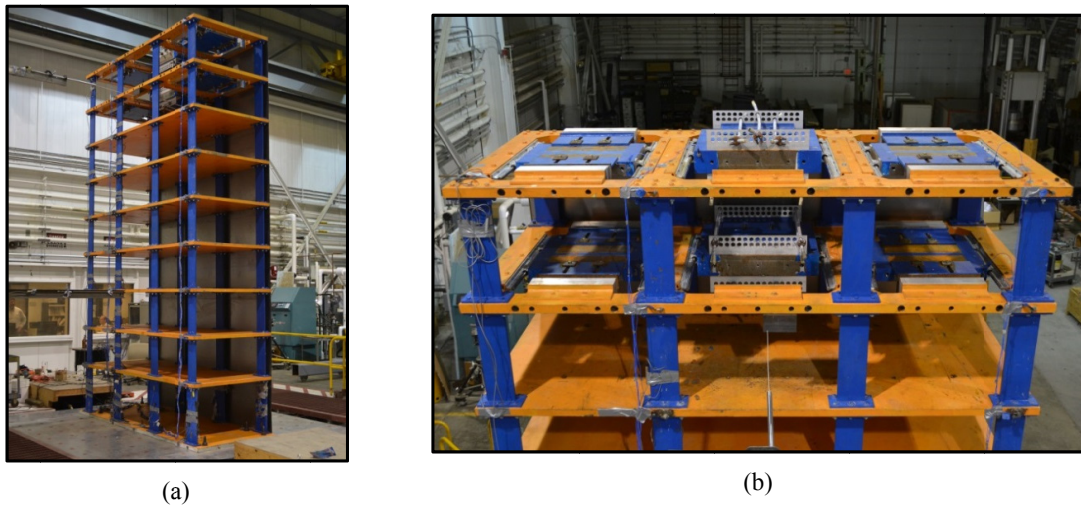


Figure 1. Base structure (a) overall view (b) close up view of top floors and NESs

2.2 System of nonlinear energy sinks

This work examines the effectiveness of a system of NESs at attenuating the response of the base structure detailed in Section 2.1 to seismic ground motion. This system of NESs features six individual NESs and is incorporated into the top two floors. As shown in Figure 1 and Figure 2, each of the top two floors of the base structure contains two Type I NESs and one single-sided vibro-impact NES (SSVI NES). Phenomenological models of these NES types are shown in Figure 2. Each of the NESs in this system has its own locking mechanism; consequently, tests could be performed with all of the NESs free to move, all the NESs locked, or part of the system of NESs locked and unlocked.

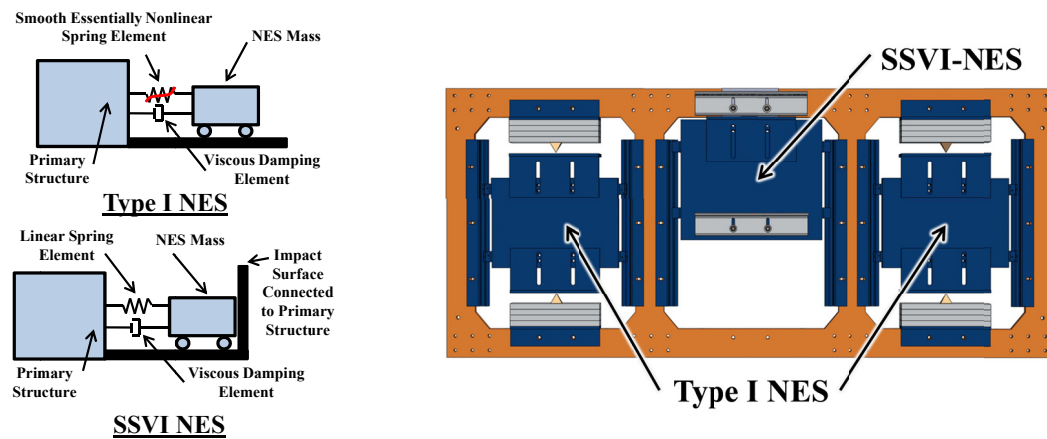


Figure 2. Phenomenological models of NESs and rendering of NESs positioned in floor plate

2.2.1 Single-sided vibro-impact nonlinear energy sinks

As Figure 2 shows, the idealized representation of the SSVI NES consists of a mass attached to a primary structure with a viscous damping element and a linear spring element; for this type of NES the relative displacement of the mass is limited on one side due to an impact surface that is connected to the primary structure. Due to the discontinuity in restoring force this creates, these impacts are broadband events. As a result of these broadband events, energy is scatter throughout the structure, including to its higher modes (Al-Shudeifat et al. 2013). The transfer of energy to higher modes that

occurs due to the addition of this type of NES is beneficial because, at these higher frequencies, the energy can be naturally dissipated faster due to the reduced time scale. Furthermore, this NES can also help to attenuate the response of the base structure due to the energy dissipated by its motion relative to the point it is attached.

The physical realization of the SSVI NESs used in this work consists of solid steel masses that have Thomson SSUPB012 pillow blocks containing linear bearings mounted on their sides. These linear bearing allow the masses to move on sets of 1.91 cm round rail that are attached to the base structure’s floor plates. The rails are positioned such that the masses moved uniaxially in the short direction of the structure. Each SSVI-NES is coupled to the floor plate with a set of elastic cords. These elastic cords are positioned such that, at rest, the NES mass is in contact with one side of the floor plate. While unintended, there was a small amount of pretension in the elastic cords; the result of this was that the NES mass was secured against the side of the floor plate under very low loading conditions. When the motion of the floor is large enough to exceed this small pretensioning force the SSVI-NES moved relative to the floor. When this happens the elastic cords connecting the NES to the floor plate stretch and produce a restoring force that results in the NES mass being forced to collide with the floor with significant velocity. The result of this steel on steel collision is energy dissipation, the amount of which is related to the coefficient of restitution, and high frequency scattering of the remaining amount of energy. The properties of the SSVI NESs examined in this work are shown in Table 1.

Table 1. NES design values

	Mass (% of Structure Total Mass)	Design Stiffness Coefficient (N/m)	Design Stiffness Exponent
8th Floor Type I NES	1.5	7.56×10^8	3
9th Floor Type I NES	1.5	1.07×10^8	3
8th Floor SSVI NES	3.5	14546	1
9th Floor SSVI NES	3.5	12219	1

2.2.2 Type I nonlinear energy sinks

As shown in Figure 2, the idealized representations of the Type I NES is composed of a mass connected to a primary structure with a viscous damping element and a smooth (no discontinuities in the restoring force) essentially nonlinear spring element. The smooth essential nonlinearity of this spring element means that the response of the NES is not dominated by a single natural frequency and the device can resonate with and participate in the transfer of energy with any mode of the primary structure (Vakakis et al. 2008). As with the SSVI NES, the transfer of energy for low to high frequencies, which is possible due to this essential nonlinearity, allows this NES to quickly attenuate the motion of a structure it is attached to.

The physical realization of the Type I NESs also took the form of steel blocks that move on bearing; however; in this case the Type I NESs are coupled to the structure using specially shaped elastomeric bumpers that are mounted on each side of the NES and are put into compression when the NES moved. The special shape of these bumpers provides an essentially nonlinear, and nearly cubic, restoring force to these NES masses. Each Type I NES on the 8th floor has the same set of bumpers, but the Type I NESs on the 9th floor have a different set of bumpers. The properties of the Type I NESs examined in this work are shown in Table 1.

2.3 Instrumental

To measure the response of system to the ground motion, accelerometers were placed on the base structure and the NES masses. Using a combination of filtering and numerical integration,

displacements can be estimated from the measured accelerations. Additionally, to measure the actual demand on the structure, strain gages were placed on the first floor columns.

3 EFFECTIVE DAMPING

The base structure presented in the previous section is not designed to yield during the seismic ground motion used in this work; however, because of the effects of the NESs, the response of the structure is nonlinear. This nonlinearity response in the base structure is apparent when examining its modal response, which contains energy exchanges between modes. Due to this nonlinear response, traditional measures used to describe the response of the structure, such as modal damping coefficients and loss factors, do not directly apply. As a consequence of this, a measure referred to as effective damping was developed (Sapsis et al. 2012). This measure examines the response of a particular structural mode, which includes the intrinsic modal damping as well as the transfer of energy in and out of the mode due to the effects of the NESs, then assigns a linear damping value that would best represent this behaviour. While this measure was initially developed to evaluate a structure's free-vibration response, this measure has been extended by (Ott, R. J. 2012) to consider the forced vibration response as well. Below, the theoretical background for this measure is briefly introduced; for a complete derivation and discussion refer to (Ott, R. J. 2012; Sapsis et al. 2012).

As effective damping is a measure that evaluates the modal response of a structure, the derivation of this measure will be done in modal coordinates. The linear mass normalized equation of motion for a particular mode, i , is

$$\ddot{u}_i + 2\zeta_i \omega_{n,i} \dot{u}_i + \omega_{n,i}^2 u_i = f_i(t)/m_i \quad (1)$$

Where u_i , m_i , ζ_i , and $\omega_{n,i}$ are the i^{th} mode displacement, mass, damping coefficient, and natural frequency, respectively. Additionally, $f_i(t)$ is the for the i^{th} mode modal forcing function. The mass normalized energy in this system can be defined as the normalized combination of the kinetic and potential energy.

$$\frac{E_i}{m_i} = \frac{1}{2} \dot{u}_i^2 + \frac{1}{2} \omega_{n,i}^2 u_i^2 \quad (2)$$

The time rate of change of the energy in this mode is defined as

$$\frac{\dot{E}_i}{m_i} = \dot{u}_i (\ddot{u}_i + \omega_{n,i}^2 u_i) \quad (3)$$

This equation can be rewritten to include the forcing function on the system by substituting in \ddot{u}_i , as defined by Eq. (1).

$$\frac{\dot{E}_i}{m_i} = \frac{f_i(t)}{m_i} \dot{u}_i - 2\zeta_i \omega_{n,i} \dot{u}_i^2 \quad (4)$$

The coefficients on the \dot{u}_i^2 in Eq. (4) can be replaced by $\lambda_{\text{eff},i}$, which is defined to be the instantaneous effective damping.

$$\frac{\dot{E}_i}{m_i} = \frac{f_i(t)}{m_i} \dot{u}_i - \lambda_{eff,i} \dot{u}_i^2 \quad (5)$$

Eq. (5) then can be rearranged to solve for $\lambda_{eff,i}$

$$\lambda_{eff,i} = \frac{\frac{f_i(t)}{m_i} \dot{u}_i - \frac{\dot{E}_i}{m_i}}{\dot{u}_i^2} \quad (6)$$

In a nonlinear system, such as the one considered in this work, $\lambda_{eff,i}$ is a time varying quantity; however, it is advantageous to define an effective damping measure that evaluates the complete time history of a response. Consequently, the global effective damping of the i^{th} mode response of a system is defined as

$$\bar{\lambda}_{eff,i} = \frac{\int_0^{T_{end}} \left(\frac{f_i(\tau)}{m_i} \dot{u}_i - \frac{\dot{E}_i}{m_i} \right) d\tau}{\int_0^{T_{end}} \dot{u}_i^2 d\tau} \quad (7)$$

To facilitate practical calculation of the effective damping in this work, the fast dynamics and noise in the response of the mode considered were eliminated by high pass filter the response. After this filtering, the slow dynamics of the system, which are primarily responsible for the system's damping, remain. Eq. (8) and (9) present the expression for the instantaneous and global effective damping using this filtering. In these equations $\langle \rangle$ denotes a low pass filtered quantity. For the calculation of the effective damping presented in this work the low filter cut-off frequency was selected as 0.75 Hz. This cut-off frequency is low enough to eliminate most of the noise in the system response as well as the fast dynamics of the mode considered, but high enough to capture relatively quick changes in the energy profile of the response.

$$\lambda_{eff,i} = \frac{\left\langle \frac{f_i(t)}{m_i} \dot{u}_i \right\rangle - \left\langle \frac{\dot{E}_i}{m_i} \right\rangle}{\left\langle \dot{u}_i^2 \right\rangle} \quad (8)$$

$$\bar{\lambda}_{eff,i} = \frac{\int_0^{T_{end}} \left(\left\langle \frac{f_i(\tau)}{m_i} \dot{u}_i \right\rangle - \left\langle \frac{\dot{E}_i}{m_i} \right\rangle \right) d\tau}{\int_0^{T_{end}} \left\langle \dot{u}_i^2 \right\rangle d\tau} \quad (9)$$

4 SHAKE TABLE GROUND MOTIONS

Testing of the structure outlined in Section 2 was performed using the Triaxial Earthquake and Shock Simulator (TESS) shake table at the US Army Corps of Engineering Construction Engineering Research Laboratory in Champaign, IL (U.S. Army Engineer Research and Development Center 2008). This large 3.66 m by 3.66 m shake table can support payloads in excess of 50000 kg. Before beginning testing, a routine developed by the manufacturer of the TESS shake table control system,

designed to improve the tracking of the shake table, was performed. In this routine, measurements of the shake table response were acquired for individual sine sweeps performed in each of the shake table's six degree-of-freedom. A compensation matrix that was developed from these measurements was then automatically applied to each ground motion commanded by the system.

Using this shake table, the structure was loaded with a scaled version the ground motion measured from the JMA Station during the 1995 Kobe earthquake. An example of the experimentally reproduced scaled version of this scaled ground motion is shown in Figure 3.

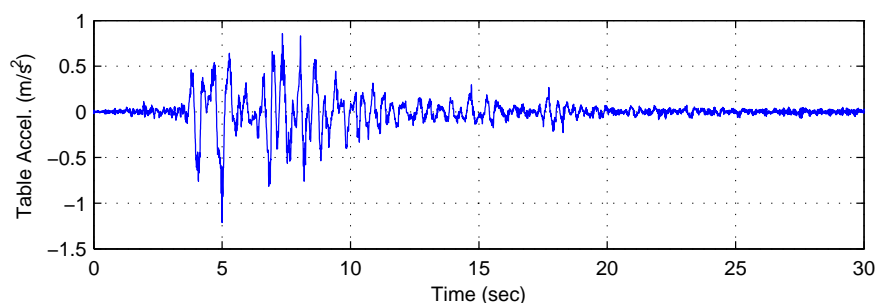


Figure 3. Example Kobe JMA scaled ground acceleration record produced by the TESS shake table

5 EXPERIMENTAL RESULTS

Using the scaled JMA station ground motion record from the 1995 Kobe earthquake, shake table tests were performed on the large-scale structure equipped with the system of NESs, which was introduced in Section 2. Tests on this structure were done for several cases including with only the Type I NESs unlocked and able to move relative to the structure, only the SSVI NESs unlocked, the whole system of NESs unlocked, and with the whole system of NESs locked. The resulting seventh floor displacement response from these tests is shown in Figure 4a, c, e, and g. As this figure shows, during cases when a portion or the entire system of NESs was unlocked, the response of the system was dramatically reduced, including a substantial reduction in peak displacement, compared to the locked case. While response reduction is seen in all cases with NESs unlocked, in some cases better performance is observed. An example of this is the superior performance, in terms of peak displacement reduction, is the structure's response with the SSVI NES unlocked compared to the case with the Type I NESs unlocked. Another example is the improved attenuation of the low level residual motion that remains after the structure's high amplitude response, which is seen in the case with the Type I NESs unlocked compared to the case with the SSVI NES unlocked. However, with the whole system of NESs unlocked, the individual aspects of the response that the Type I and SSVI NES excel at mitigating are combined to create a system that shows the best overall performance.

Wavelet spectra are used to examine the time dependent frequency content of the structure's response. The wavelet spectra corresponding to the acceleration response of the structure's seventh floor when subjected to the ground motion is shown Figure 4b, d, f, and h. This figure shows that the frequency content of the structure's response is concentrated around the first couple structural modes when all the NESs were locked; however, when the NESs were unlocked there is a dramatic shift in frequency content of the response to the higher modes and a rapid attenuation of the lower mode response. This shift in frequency content is evident when the Type I NESs were unlocked, but it is far more evident when the SSVI NESs and the whole system of NESs were unlocked due to the strong impacts of the SSVI NESs.

The first floor columns of a structure are often times where the peak demand on a structure occurs; consequently, to examine the demand on the structure, strain in the first floor columns was measured. Figure 5 shows the time history of the strain in the columns when subject to the ground motion for the cases when the Type I NESs are unlocked, the SSVI NESs are unlocked, all the NESs are unlocked, and all the NESs are locked. This figure shows that, in all of the cases when the NESs are unlocked, significant attenuation of the response occurs compared to the locked case. This attenuation includes a reduction in peak strain, which is observed to be most significant in the cases when the SSVI NESs are unlocked or all the NESs are unlocked.

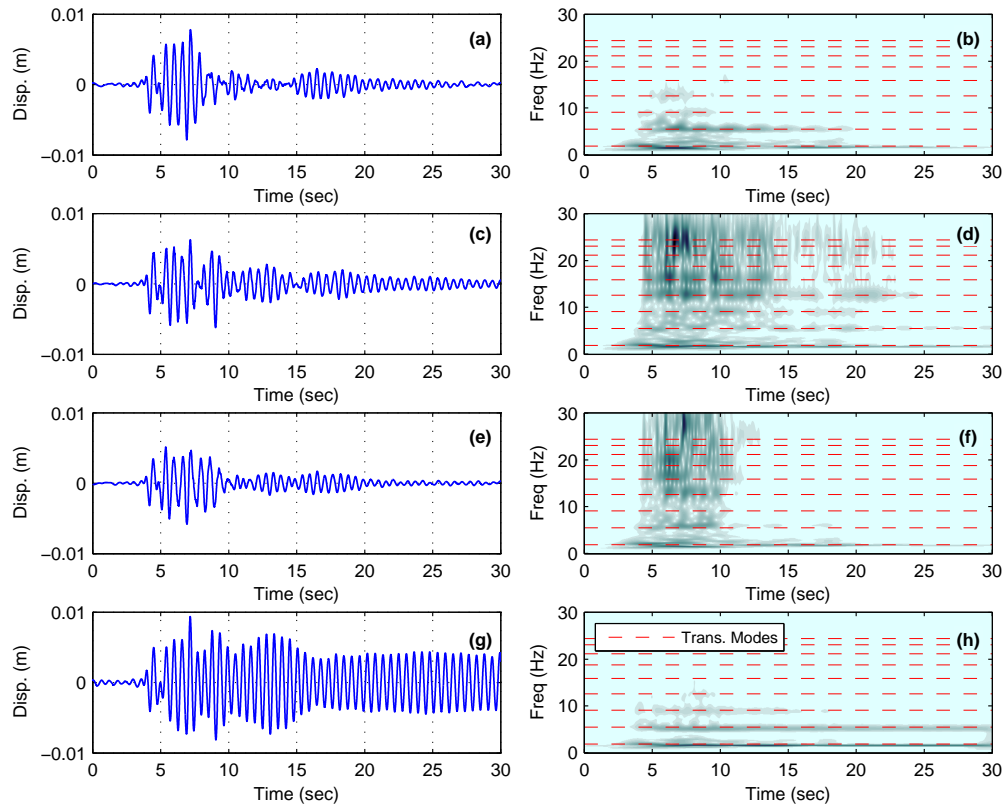


Figure 4. Seventh floor displacement (a) Type I NESs unlocked, (c) SSVI NESs unlocked, (e) All NES unlocked, and (g) All NES locked; seven floor acceleration wavelet spectra (b) Type I NESs unlocked, (d) SSVI NESs unlocked, (f) All NES unlocked, and (h) All NES locked;

To examine how the amplitude of the ground motion affects the performance of the system of NESs, shake table tests were performed with multiple different scaling levels of the ground motion shown in Figure 3. As discussed in Section 3 the apparent damping in the response of the structure can be quantitatively examined through calculation of the effective damping; consequentially, the system's amplitude dependency will be investigated through examination of the effective damping. Figure 6 shows the change in 1st mode effective damping across a range of ground motion amplitudes for each of the NES cases. As expected, this figure shows that with the NESs locked very low 1st mode effective damping is consistently measured. Additionally, this figure shows that the peak effective damping of the case with the Type I NESs unlocked and the case where the SSVI NESs are unlocked occurred at different ground motion amplitudes; as intended, the peak of the Type I NES occurs at a lower amplitude and the peak of SSVI NES case occurs at a high amplitude. Furthermore, the synergy between the types of NESs is demonstrated in the case with all NESs unlocked as this case shows a high level of 1st mode effective damping that is relatively consistent across all ground motion amplitudes considered.

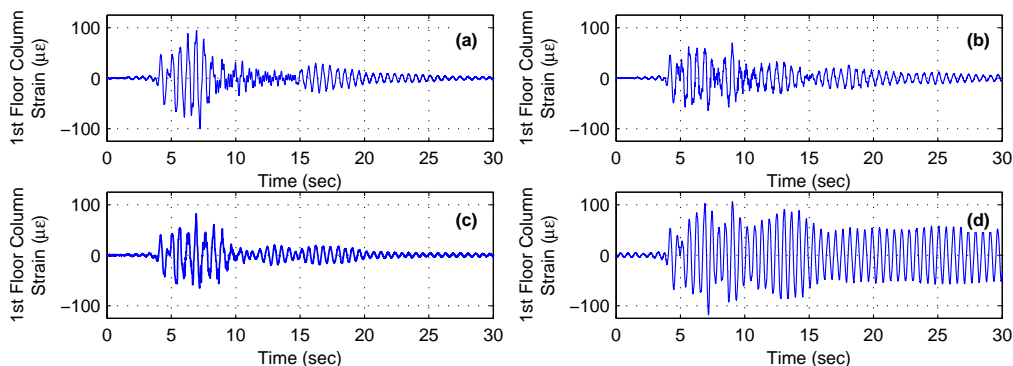


Figure 5. 1st floor column strain (a) Type I NESs unlocked, (b) SSVI NESs unlocked, (c) All NES unlocked, and (d) All NES locked

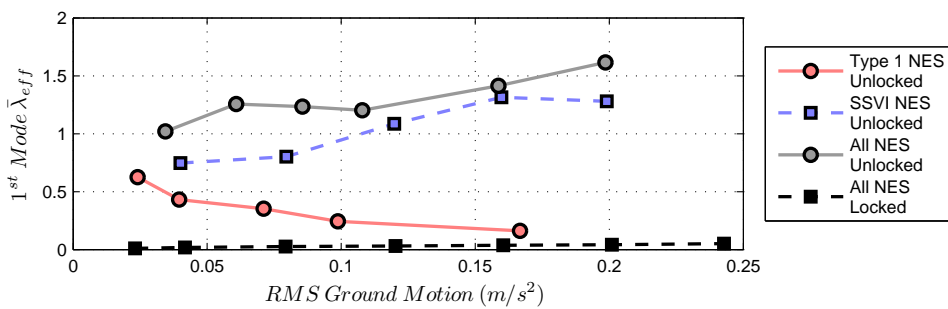


Figure 6. 1st mode effective damping

6 CONCLUSIONS

In this paper, the effectiveness of a system of nonlinear energy sinks (NESs) at controlling the response of a large-scale base structure subjected to seismic ground motion was experimentally evaluated. The system of NESs examined in this work consists of a combination single-sided vibro-impact (SSVI) NESs, which utilize a non-smooth essential nonlinearity, and Type I NES, which utilize a smooth essential nonlinearity. Examination of the response of the structure to a scaled version of the 1995 Kobe earthquake show that improvements to the attenuation of the structure’s response, including a reduction in peak demand, can be observed when the entire system of NESs is unlocked as well as when only the Type I or SSVI NESs are unlocked. Furthermore, tests across a range of ground motion scaling factors indicated that the peak effectiveness of the Type I NESs and the SSVI NESs occurred at different ground motion amplitudes; however, due to the synergy between the types of NESs, superior and more constant performance was observed across a large range of ground motion amplitudes when the entire system of NESs was unlocked.

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